Should you buy or sell tail risk hedges? A filtered bootstrap approach

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Abstract

The 2008 financial crisis has spurred investors to wonder about adding tail risk hedges to their portfolios. However, the cost of these hedges can often be a deterrent. As a consequence, many financial institutions have tried to develop cost-effective products for investors who wished to protect against tail risk. In particular, dynamically buying protection, i.e. selling a previously bought hedge when its market value is high enough, seems to remarkably reduce the cost of being protected. Some academicians and practitioners have long argued that investors shall instead sell tail risk hedges, so to cash in their substantial premia. Thus, in order to understand if there is an optimal approach to tail risk protection, we have looked at five competing strategies which are differently involved with tail risk hedges, and by relying on a semiparametric approach based on filtered bootstrap we have compared their return distributions. We show that there can hardly be a unique strategy suitable for all investors. For instance, our results suggest that selling protection and waiting till the maturity of the hedge can easily outperform a strategy which also sells the hedge but buys it back before this expires. This solution, however, probably suits those investors that, rather than hedging against tail risk, are actually ready to be more exposed to tail events, in order to boost portfolio returns. Conversely, dynamically buying protection may indeed answer the needs of those investors who are willing to hedge against tail risk in a cost-effective manner.

Introduction

Since the outburst of the financial crisis in 2008, many investors have started wondering whether it would be appropriate to protect their portfolios from "tail risk", i.e. the risk of suffering extreme losses. Such losses, especially before the crisis, have often been regarded as almost impossible. But events such as the Lehman Brother's default, or more recently the European sovereign debt crisis, have shown that these events are actually more likely than it was commonly thought.

In response to this new awareness, the number of asset managers proposing tail risk hedging strategies has increased. However, investors have soon realized that the cost of implementing these strategies can be significant, especially during times of market dislocation. The industry has thus responded by focusing on cost-effective tail risk protections. Yet many investors still prefer to bear the tail risk rather than commit to the periodic payment of a considerable insurance-like premium.

This work attempts to understand whether and how an investor can enhance the return distribution of her portfolio by hedging the tail risk or by selling protection against it. Namely, by employing the filtered bootstrap technique, we will obtain the return distributions associated with five portfolio strategies which are differently involved with tail risk hedging. These distributions will then be compared by looking at their main statistical moments, as well as by analyzing each strategy's exposure to extreme losses. To motivate this study, the next section briefly reviews some of the main research contributions to this ongoing debate.

Literature Review

Literature review on Filtered Bootstrap

In order to obtain the return distributions for the various strategies we take into consideration, we have relied on the filtered bootstrap technique, proposed by Barone-Adesi, Giannopoulos and Vosper (1999). This technique has been widely applied within the context of VaR (see for instance Marsala, Pallotta and Zenti (2004) for a brief review of this technique, or Chrétien and Coggins (2010) for a comparison of filter-based VaR relative to other VaR approaches). Interestingly, the filtered bootstrap approach also seems to deliver quite an accurate option pricing performance, as shown in Barone-Adesi, Engle and Mancini (2008).

Literature review on Tail Risk Hedging

One of the main areas of research in tail risk hedging debates on agents' risk preferences towards tail events, and on whether tail insurance is overpriced in the market. For instance, Taleb (2004) argues that agents underestimate the probability of large adverse shocks. Similarly, Bhansali (2009) states that many investors underrate the true risk posed by tail events, and he therefore maintains that tail risk insurance is not an extravagant optional for the portfolio, but rather a necessary protection. From a different perspective, on the basis of systemic considerations concerning the whole financial system, Litterman (2011) argues that buy-side investors should sell tail risk protection to banks, rather than buying protection from the latter. On the same side of the debate, Illmanen (2012) argues that insurance (limiting the left tail) and lottery tickets (enhancing the right tail) are usually overpriced, therefore selling them can boost long-term returns. However, Taleb (2013) has recently tried to refute this conclusion by Illmanen, by pointing out that the return of OTM puts in case of extreme losses displays convexity to distance from moneyness, and that OTM option's payoff is convex to implied volatility.

A parallel stream of research on tail risk hedging explores cost-effective ways of implementing protections against tail events. For instance, McRandal and Rozanov (2012) describe the main types of tail risks, and highlight the benefits of an active, diversified approach to tail risk hedging. Miccolis and Goodman (2012) likewise propose an "integrated" form of tail risk protection, whereby dynamic asset allocation is combined with tail risk hedges. Finally, Bhansali and Davis (2010a and 2010b) argue that tail risk hedging shall not be conceived as a "sunk cost", but rather in an "offensive" fashion, by actively exploiting the spikes that the value of the hedge may experience throughout its life.

The Strategies

Description of the strategies

In order to understand whether and how tail risk hedges may be of some advantage to the investor, we have compared five different types of portfolio strategies. This comparison has been carried out over three investment horizons: the short-term (1 year), the medium-term (5 years) and the long-term (15 years). Each of the five strategies is defined by three parameters: the budget that the investor is ready to put aside for tail risk hedging overlays, the maturity of the options bought or sold and, for the dynamic strategies, the "monetization multiple". The latter is defined as the number of times the market value of the option needs to be greater (smaller) than the value the option had when the investor bought (sold) it, in order for the dynamic strategy to sell (buy) the option. In order to streamline the analysis, we have decided to limit the discussion to two possible values for each of these three parameters. In doing so, we have chosen relatively extreme values, aiming at maximizing the differences. Namely, the budget can be 50 bps or 150 bps, the option maturity can be 3 month or 12 month, whilst the monetization multiple can be either 0.5 or 5.

- **Buy And Hold on the S&P500**. It buys today the S&P500, and then passively waits till the end of the investment horizon. The portfolio return is just going to be the return on the S&P500 over this horizon. We will take this strategy as a benchmark against which to compare the remaining four strategies.

- **Static Purchase**. At the inception of the investment horizon, this strategy buys the S&P500 with weight $(1-b)\%$, and buys one OTM European put option on the same index with weight $b\%$, where b is the budget the investor is ready to spend in order to protect its portfolio from tail risk. The budget and the maturity of the option are assumed not to change throughout the whole investment horizon. Also, the put in the portfolio is never sold before maturity. When the option expires, the hedge is rolled over: the strategy buys a a new put whose premium is $b\%$ of the market value of the whole portfolio on the day of the hedge rollover. Given that the budget is fixed, the "attachment point" of the hedge (i.e. the moneyness of the put) will depend on the market conditions when the hedge is rolled over. Namely, the worse the performance of the S&P500, the more OTM the new put which replaces the hedge just expired. This happens because the volatility is likely to have increased as a result of equity market distress, therefore propping up valuations across the whole option chain.

  If at maturity the option is ITM, then it is automatically exercised, and the cash generated will be used to roll over the hedge up to $b\%$ of the current market value of the portfolio. Any extra cash will be invested in equity at current valuations. In case the cash delivered by the exercise is not enough, any uncovered amount will be financed by selling the S&P500 at current valuations. If instead at maturity the option is ATM or OTM, the whole cost of the new hedge will be financed by selling part of the current equity position. If upon a certain rollover date market conditions are bullish enough to make the premium of all the OTM puts lower than $b\%$ of portfolio's market value, then the strategy does not overhedge, but rather buys an ATM option and the remaining available budget is left in the equity position.

- **Static Sale**. At the inception of the investment horizon this strategy buys the S&P500 with weight $(\frac{100}{100+b})\%$, and sells one OTM European put option on the same index with weight $b\%$. In this case, b can be interpreted as a risk budget: the higher the premium received, the less OTM the option, so the easier it will end up ITM at maturity, resulting in an a loss for the investor. Similarly to what happens for the Static Purchase, the option position is never closed before maturity. The whole premium is kept as cash in the margin account throughout the whole life of the option. Differently from the case of a long position in the option, this assumption slightly underestimates the margin requirement actually set by CBOE[1], so that in analyzing the mean return of this strategy we shall bear in mind that it may be moderately overestimated. At maturity, the option overlay is rolled over: a new put option is sold such that its premium is $b\%$ of the market value of the whole portfolio on that day. Of course, the moneyness of the put sold will depend on the market conditions when the option position is rolled over.

  If at maturity the option is ITM, any due amount in excess of the premium initially collected will be financed by selling part of the equity position. If instead the current margin account is enough to meet the obligation towards the holder of the option, any residual amount in the margin account will be used to increase the equity exposure. If instead at maturity the put is ATM or OTM, the whole cash initially collected is invested in the equity market. In case upon a certain rollover market conditions are bullish enough to make the premium of all the OTM puts lower than $b\%$ of the current market value, then the strategy does not sell an ITM option just to guarantee a premium that would be $b\%$ of the market value of the portfolio, but it rather collects a lower premium.

[1]The margin requested by the CBOE for short positions in broad index options shall include not only the whole premium of the option (or its market value, once the position has been opened) but also 15% of the price of the underlying in case the option is ITM, or the maximum between 10% of the strike and the difference between the strike and 85% of the price of the underlying in case the option is OTM.

- **Dynamic Purchase**. This strategy takes inspiration from the proposal put forward in Bhansali, Davis (2010a) and Bhansali, Davis (2010b). The basic structure is similar to the "Static Purchase" strategy. However, a dynamic element is introduced. Given a "monetization multiple" m, if on any day before maturity the market value of the put option held in the portfolio exceeds m times its initial cost, then the put is sold. The cash generated by this monetization, for an amount equal to $b\%$ of the market value of the portfolio on the day of monetization, is used to replace the hedge. The new put bought will have the same maturity as the one just sold (by construction), but it will probably be more OTM than the latter, given the market instability which has fuelled the valuation of the formerly held put. Any remaining amount of the monetization proceeds is invested in the equity market, since upon monetization equity valuations are likely to be attractive[2].

- **Dynamic Sale**. The basic structure is similar to the one of the "Static Sale" strategy. However, a dynamic element is introduced. If on any day before maturity the market value of the put option is $\frac{1}{1+m}$ times the value of the premium initially collected, then the option position is closed by buying back the same option[3]. The cash needed to finance this purchase is taken from the margin account. Any residual amount held in the margin account is invested in the equity market. Then, a new option with the same maturity and the same premium is sold, but it will most likely have a higher strike than the one just bought back, as the S&P500 is likely to be richly priced, and the volatility is probably hovering around low values[4].

The dynamic equity beta of the strategies

When comparing the strategies it is important to bear in mind that, a part from the Buy and Hold on the S&P500, all the other strategies have an equity beta (i.e. an exposure to the S&P500) which varies through time. Namely, the Static Purchase strategy will tend to have an equity beta lower

[2]For this strategy, we have assumed that over the 1-year investment horizon the only available option maturity is 3 month, given that resorting to 12 month options would imply buying protection beyond the 1-year horizon every time there is a monetization.

[3]For consistency, we will refer to level of the option's market value triggering a dynamic behavior as "monetization multiple" also in this case, although here it is not particularly appropriate to talk about monetizations.

[4]Similarly to the Dynamic Purchase, for the Dynamic Sale strategy we have assumed that over the 1-year investment horizon the only available option maturity is 3 months.

than one, because by construction it constantly hedges part of the equity exposure (upon each hedge rollover, for a given budget the magnitude of the hedge will depend of course on the market performance of the S&P500 at that time). The beta of the Static Purchase strategy will increase only if the put option bought is ITM at maturity. Similarly, the Dynamic Purchase strategy will have an equity beta aligned with that of the Static Purchase if the S&P500 trends upwards, but its beta will increase upon every monetization. Conversely, the Static Sale strategy tends to have an equity beta higher than one, since it constantly sells put options on the S&P500, thus increasing its exposure to any move of this index. The beta of the Static Sale strategy will decrease only if at maturity the option sold it ITM. Finally, the Dynamic Sale strategy will have an equity beta aligned with that of the Static Sale, but its beta diminishes every time the put option initially sold is bought back.

In fact, an alternative approach in constructing the strategies might have been to target a constant equity beta of one for all the five strategies considered, so that the strategies take on about the same level of risk. This would translate into rebalancing the underlying exposure to the S&P500 upon rolling over the options, so that the overall equity beta of the strategy is one, therefore coinciding with that of the Buy and Hold on the S&P500. For instance, an investor wishing to implement the Static Purchase strategy would need to increase her investment in the equity market every time she rolls forward the put option bought as a hedge, in order to achieve an equity beta of one. Similarly, an investor implementing the Static Sale strategy would need to scale down her investment in the equity market every time she sells new put options.

However, we have chosen not to incorporate the equity beta in the periodic rebalancing of the strategies, given that we believe it would be more likely for a standard investor just to size the put options bought or sold on the basis of its budget available, without scaling up or down the underlying equity exposure to maintain a constant equity beta.

A closer look at dynamic strategies

The Dynamic Purchase strategy and the Dynamic Sale strategy can be regarded as being the mirror image of each other. In order to gain a better understanding, let us look at the risk, the cost and the benefit of each of them.

The *risk* of selling a previously purchased put before maturity is that, after the monetization, the market value of the option keeps increasing (so that we would have been better off by waiting longer). The *cost* of selling before maturity is that the new put is likely to be restriked at a lower protection level than the one just sold. However, the *benefit* of this approach is to offset the cost of tail risk protection at least partially. In other words, we temporarily give up part of the protection in order to reduce the long-term cost of the hedge.

For the Dynamic Sale strategy, the *risk* of buying back the put when its value is $\frac{1}{1+m}$ of the premium is that at maturity the option might well end up ATM or OTM, and therefore have a null value. In this case, we would be cashing in a profit that is smaller than the maximum achievable. The *cost* of closing the short option position before maturity is being exposed to a higher strike: the maturity and the available budget for the new option sold are the same ones of the put just bought back, but the bullish equity market and/or the time decay will dampen the premium of most put options, even those being just slightly OTM. In fact, differently from the Dynamic Purchase, the Dynamic Sale almost always monetizes before maturity: regardless of the equity market performance, the time decay effect will eventually bring the price below $\frac{1}{1+m}$ of the initial premium, regardless of the magnitude of m (of course, the bigger m, the more aggressive the strategy, and the closer to maturity the day of the anticipated rollover). Finally, the *benefit* of this strategy is to reduce the risk that unexpected spikes in volatility might bring the option to end up ITM at maturity, which would result in an outflow for the portfolio.

The Simulation Procedure

The Filtered Bootstrap model

In our attempt to understand which use of tail risk hedges can make the investor better off, we have tried to go beyond the usual historical approach used in previous studies of this kind. Indeed, the common procedure compares the mean cumulative return for different portfolio strategies over a certain historical sample. No doubt this solution is tempting: it is relatively simple, and it avoids having to make strong assumptions on the distribution of market returns (except assuming it is stationary, of course). Nonetheless, one of its main drawbacks is that it only delivers information on the mean of the distribution of historical returns. In order to have a more solid understanding of the properties and the risks associated with a given strategy, we feel that looking exclusively at the mean return may be misleading. As a consequence, we have tried to obtain an estimate of the whole return distribution.

True, following the common framework just described, we could have looked at how a certain strategy would have behaved day-by-day in the past history, and we would have thus obtained an estimate for the return distribution of that strategy. Even so, the best we could have dreamt for would have been a distribution for returns that are defined on a daily, weekly or monthly basis. Let us suppose we need a distribution for annual returns: in the best case scenario, we could have had almost one hundred annual historical returns for, say, the American equity market, but we would have anyway been constrained by the length of the time series of liquid option prices. This drawback is obviously exacerbated if we look at horizons longer than twelve months. Unfortunately, discussing the impact of tail insurance policies by only looking at investment horizons which are shorter than a year would have been seriously inadequate.

To solve this issue, a popular approach is to use some bootstrap techniques such as the "moving block bootstrap", which can however be prone to considerable estimation risk (see Cogneau, Zakamouline (2010)). An alternative solution would be to infer option prices on those segments of the historical sample for which no liquid quotes are available. For example, this is the choice of Bhansali and Davis (2010b), who employ a Threshold Factor Augmented VAR model. Arguably, this solution reintroduces some form of model risk, which the pure historical analysis was trying to minimize.

Given that estimating the return distribution for investment horizons longer or equal than a year seems to require some kind of modeling structure, we have thought that the filtered bootstrap technique could have been a sensible solution. In fact, it potentially allows to obtain as long a time series as we like. Of course, we would need the observations to be independent; for this aim, we have chosen a GJR-GARCH(1,1) specification as a filter for the historical return observations on the S&P500 (see Glosten, Jagannathan and Runkel (1993)):

$$r_t = \sigma_t \epsilon_t$$

$$\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \theta I_{t-1} r_{t-1}^2 + \beta \sigma_{t-1}^2$$

where r_t is computed as log return, and I_{t-1} is 1 if $r_{t-1} < 0$ and 0 otherwise (the other variables have their conventional meaning)[5]. Given that we adopt a bootstrap approach, we don't make any parametric assumption on the distribution of ϵ_t. Clearly, this is also where the modelling risk kicks in. However, we have favored the GJR-GARCH because, as shown by Engle and Ng (1993), it well describes the news impact curve, and because it can outperform many competing GARCH models, as highlighted by Rosenberg and Engle (2002). Besides, when empirical innovations are used, it also delivers reliable estimates for option prices, as long as the volatility forecasts are produced on the basis of risk-neutral state price densities (see Barone-Adesi, Engle and Mancini (2008)). Also, a further assumption of the filtered bootstrap framework is the stationarity of the underlying distribution of market returns. In point of fact, the GJR-GARCH is calibrated on historical data, and through the filtered bootstrap approach we are implicitly assuming that the historical observations represent a reliable description of future returns.

The estimation algorithm

This section explains the steps we have followed in order to calibrate the model. The next section deals instead with the procedure we have adopted in order to simulate the return distribution for each strategy.

[5] Since on daily frequencies the mean component of the returns is negligible, we have preferred to assume a zero mean return. On top of that, modeling the conditional mean would have implied at least another parameter to be estimated, therefore adding an unnecessary level of complexity.

1. **Estimation of the GJR on the basis of the historical state price density**. The first step has been estimating the parameters of the GJR starting from the historical S&P500 log returns. We have looked at the returns from January 2003 to December 2012. The QMLE minimization has produced an estimate for each of the four parameters of the GJR (ω = 1.5667E-6, $\alpha = 0$, $\theta = 0.1330$ and $\beta = 0.9177$). The minimization has also delivered estimates for the standardized residuals, for which have the Ljung-Box routine has confirmed serial independence[6]. The values of the four parameters, together with the series of standardized residuals, have then been used in a later step to simulate the prices for the S&P500.

2. **Obtaining GJR option prices**. The GJR parameters estimated on the basis of the historical data cannot also be used to simulate the prices of the put options written on the S&P500. Indeed, several studies (see, for instance, Chernov and Ghysels (2000), or Christoffersen and Jacobs (2004)) show the poor pricing performance of options valuations based on a volatility process which is calibrated on historical state price densities. We therefore needed to run a second calibration of the GJR, this time on the risk neutral state price density. Operationally, we have first taken historical implied volatilites of European OTM puts on the S&P500 on each trading day of the sample. Our option chain featured four maturities (1 month, 3 month, 6 months and 12 months), and eleven levels of moneyness (equally spaced from 50% OTM to the ATM level). Such volatilities have been converted into historical prices. Meanwhile, we have computed the option prices on the same dates of the sample via Black-Scholes: the volatility process was the one produced by the GJR with historical innovations, the prices of the underlying were the historical values of the S&P500, and the strikes were determined by our eleven levels of moneyness.

3. **Estimation of the GJR on the basis of the risk-neutral state price density**. We have then minimized the MSE (mean squared error) on each day of our 10-year sample, where the error has been defined as the difference between the true historical price of the options and the price whose volatility was based on the GJR. The minimization routine produces, for each

[6]Given that the bootstrapping routine does not make parametric assumptions on the standardized residuals, the MLE procedure becomes a Quasi-MLE. The estimates are still consistent as long as the mean and variance models are correctly specified.

of the four parameters of the GJR, as many estimates as the days in the sample. The risk neutral estimate of each parameter has been computed as the mean of all its daily estimates (returning $\omega = 3.5056\text{E-}6$, $\alpha = 0.0022$, $\theta = 0.2543$ and $\beta = 0.8675$)[7]. Through these risk neutral estimates of the parameters of the GJR, we have filtered the historical returns of the S&P500, and we have obtained the risk neutral innovations. These innovations have been standardized and checked for independence via the Ljung-Box test.

The simulation algorithm

Once we have obtained the historical estimates as well as the risk neutral estimates of the parameters of the GJR, we have moved to the second phase of the procedure, which was about simulating the price process for the S&P500 and for the corresponding put options over the desired investment horizons (1 year, 5 years and 15 years). In order to streamline the analysis, we have limited the available option maturities to the 3 months and the 12 months expiries.

4. **Simulation of the prices of the S&P500**. The values of the four parameters of the GJR obtained in step 1 have been used to simulate the dynamics of the volatility of the S&P500 returns. The innovations have been generated by bootstrapping (i.e. randomly sampling with replacement) the standardized residuals which were also estimated in the first step. Given our assumption of zero mean return, the simulated process for the volatility was enough to get a simulated path for the S&P500 returns, which has then been converted into a path for S&P500 prices.

5. **Simulation of the prices of the puts written on the S&P500**. The standardized residuals based on the risk neutral state price density have also been bootstrapped, and they have been plugged into the GJR calibrated through the MSE procedure. The volatility process therefore obtained has been used to generate option prices via the Black-Scholes formula. The S&P500 prices were the ones simulated in the previous step. We were therefore able to simulate a full path for the whole option chain.

[7]This procedure is similar to the one adopted in previous studies, such as Barone-Adesi, Engle and Mancini (2008), Engle and Mustafa (1992) or Christoffersen, Jacobs, Ornthanalai and Wang (2006).

6. **Computing the final values of the portfolio according to each strategy**. Based on the simulated path for the S&P500 and for the option chain, we have been able to compute the final value of the portfolio for each one of the strategies we have taken into account.

7. **Repeating the simulations several times**. Finally, the last three steps have been repeated 300 times, in order to obtain a return distribution for each strategy.

The results

Conditional on the assumptions laid out and on the validity of the model, we are going to summarize the results of our analysis according to the following three directives. First, understanding whether among our five competing strategies there is one which strictly dominates the others in terms of desirable distributional properties, or if at least we can narrow down our choice. Second, understanding whether the distributions are such that the role of the individual risk preferences is marginal in deciding whether and how to implement tail risk hedging overlays, or if instead there is by far no universal solution and the choice necessarily boils down to the investor's utility function. Third, cautiously analyzing whether our results can tell us anything on the alleged market overpricing of the tail risk premium.

As anticipated, we have considered two possible values for each of the parameters which define a certain strategy (the budget, the maturity of the options employed, and the monetization multiple). However, this choice simply derives from the willingness to avoid results which are valid only for a specific combination of parameters' values. Consequently, in the following discussion, rather than delving into how different values of the three parameter can tweak the distribution of a certain strategy, we will try to focus on the macro dissimilarities across the strategies. As a quick summary of our results, we can consider the case in which the budget for tail risk hedging programs is 50bps, the option used has a 3-month maturity and the monetization multiple is 50%. We have chosen this particular combination as it is rather representative of the general picture emerging from our analysis. We report in Appendix B the plots[8] of the densities of the return distributions to have a global view on the strategies' behavior, as well as the corresponding QQ-plots in order to gain a

[8]These plots have been obtained through kernel density estimators.

more precise insight on their tail behavior.

Each strategy has been compared against the Buy&Hold on the S&P500, which we have chosen as benchmark strategy. The results of the comparison are expressed in terms of first four moments of the return distributions, and they are reported in Table 1, Table 3 and Table 5, which refer to the short-term, the medium-term and the long-term, respectively. A more detailed analysis of the left tail of these distributions is instead contained in Table 2, Table 4 and Table 6, which refer again to the short-term, the medium-term and the long-term investment horizons. All the tables are in the Appendix.

Buy dynamically or sell statically

By looking at the moments of the return distributions and at their left tail behaviors, our results seem to suggest that in most scenarios the Static Purchase and the Dynamic Sale strategies are dominated by the Dynamic Purchase and the Static Sale strategies, respectively. These two dominance relations are indeed two faces of the same coin, and can thus be explained as a result of the same phenomenon. Namely, selling a put option when its market value is high enough can be seen as increasing the number of favorable states of the world for this strategy. Equivalently, within a short vega approach, buying back the put option before maturity often underperforms a strategy that instead waits till expiration because in most instances the time decay would bring the option to be OTM at maturity.

Although rather infrequently, there can however be some cases where an investor may prefer the Static Purchase or the Dynamic Sale strategy over the other alternatives. For instance, over the long-term the Static Purchase can leave the probability of tail events untouched, while slightly but consistently reducing their magnitude, with almost zero sacrifice for the mean return. Instead, even though we don't have enough tail observations to draw sensible conclusions on the average long-term tail loss for the Dynamic Purchase, we can anyway observe from our results that cases when the latter experiences harsher losses than its static equivalent are not impossible. Hence, a particularly risk averse investor who has a very long investment horizon may prefer not to monetize her hedges, although this heavily penalizes the mean return. Likewise, over the 1-year horizon the Dynamic Sale strategy offers the highest mean returns, therefore an investor with rather a short investment horizon and who is ready to increase her tail risk exposure may potentially be better off by dynamically selling put options.

Conditional on our assumptions, it seems that if the investor's goal is to insure against tail risk, she is probably better off by monetizing the gains offered by the spikes experienced by the value of the puts. Instead, if her goal is to boost the mean return of the portfolio, she is probably better off by waiting till the maturity of the put option sold.

Different strategies for different risk preferences

Excluding some unlikely cases where the Static Purchase or the Dynamic Sale strategies might represent the optimal solution for some type of agents, the typical investor's choice concerning whether and how implementing tail risk hedging overlays may therefore be restricted to three alternatives: passively selling put options and rolling them over at maturity; buying put options and selling them any time their market value reaches a certain multiple of the purchase price; or simply deciding to leave the portfolio as it is.

Even if the Static Sale and the Dynamic Purchase seem to have converging distributions in the long-term, they anyway answer different needs. Hence, in order to sort out which strategy among these three could best enhance the properties of the portfolio's return distribution, we would probably need to know the investor's utility functions (although including them is beyond the extension of this work, further research could refine our framework and rigorously incorporate utility functions in the analysis). In other words, according to our model, there seems to be no universally valid answer to the debate over whether it is better to protect the portfolio from tail risk, to leave the portfolio as it is, or to actually sell tail risk protection.

For instance, whilst the Static Sale strategy may be more appropriate for a hedge fund, the Dynamic Purchase seems to be the natural solution for a typical buy-side institutional investor (e.g. a pension fund) willing to protect the portfolio from tail risk without having to give up too much in terms of mean return.

Is the tail risk premium overpriced?

The primary goal of our work was twofold: first, understanding whether any of the considered strategies outperforms/underperforms the other ones in terms of properties of its return distribution; second, finding out whether it can thus make sense for an investor to buy/sell tail risk hedges. The answers to both these questions have been elaborated in the previous two sections. Instead, understanding whether the tail risk premium is overpriced was not a main objective of our analysis. However, we can still briefly describe what our framework suggests on this point.

According to our assumptions and to our model, we don't find enough evidence to conclude that the premium for tail risk protection is too high in a risk-neutral setting, as instead stated, for instance, by Illmanen (2012). In fact, should the premium of tail risk protection be overpriced (i.e. should the premium be higher than the expected tail loss), we shall probably observe a significantly lower long-term[9] mean return for the Static Purchase strategy compared to the one of the Buy&Hold portfolio. However, we have seen that the two mean returns are on average aligned over the long-term.

To reiterate, our framework has not been constructed with the primary goal to assess potential market mispricings of the tail risk premium. In fact, in order to carry out a more targeted study in this respect, one should probably extend the analysis to more than just two option maturities. In addition, a rigorous framework would probably take into account all the possible reasons for which investors buy OTM and DOTM put options, whilst in our analysis we have only looked at one of these reasons, that is the willingness to hedge tail risk. Finally, studies of this kind shall rely on the typical number of simulations used in pricing problems, which is significantly higher than the amount of scenarios we have used to estimate the return distributions of the strategies.

[9] It is probably more appropriate to carry out such an analysis over the long-term, since tail risk can more thoroughly materialize over long horizons.

Conclusions

Through our analysis, we have tried to understand whether and how systematically purchasing or selling tail risk protection can modify the properties of the return distribution of a standard investment portfolio.

In order to achieve this goal, we have taken as benchmark portfolio a Buy&Hold on the S&P500. We have then compared it against four different strategies, which can all be seen as a Buy&Hold on the S&P500 plus an option overlay. This option position could consist in passively buying put options and holding them to maturity, or selling them and equally waiting till their expiration, or alternatively purchasing put options but selling them any time their value becomes high enough, or finally selling them in the first place and buying them back when their value is small enough.

We have assumed that returns on the S&P500 can be modeled via a GJR-GARCH(1,1), where the standardized residuals are bootstrapped according to the technique described by Barone-Adesi, Giannopoulos and Vosper (1999). Therefore, after having calibrated the GJR on the historical returns of the S&P500, we were able to obtain several simulations of the returns for the same index over 1 year, over 5 years and over 15 years. We have then recalibrated the GJR on the basis of historical put option prices, and used the new estimates of the parameters to obtain just as many simulations for the implied volatility process over the same horizons considered for the S&P500 returns. The simulated S&P500 returns were transformed into prices, and the simulations for the implied volatility were used to obtain the option prices. On the basis of the S&P500 prices and the put option prices we have computed the final values for each strategy over the three different horizons. Such values have eventually be translated into return distributions.

The comparison among the strategies' distributions has shown that, conditional on our assumptions and on our model, there seem to be only few instances where the distributional properties of passively buying tail risk hedges may be preferred over those associated to a dynamic purchase of the same hedges. Conversely, selling tail protection seems on average to perform better if the put options sold are not bought back before maturity. Nevertheless, we cannot univocally conclude whether the investor is better off by leaving the portfolio without any tail risk overlay, or by adding a dynamic protection-buying strategy, or maybe by selling put options and waiting till maturity. The answer to this question probably needs the knowledge of the specific form of the utility function for each single investor (which goes beyond the scope of this work). For instance, for an investor that wants to protect from tail risk in a cost-effective way, our simulation exercise hints at the fact that she may likely be better off by buying put options and selling them during volatility spikes. Instead, for somebody ready to take on more risk and seeking higher returns our results apparently indicate that she should probably sell put options and roll them over at maturity.

As a final note, we shall underline that our framework can be enhanced and extended in many aspects. For instance, further developments could refine the conditional volatility specification we have assumed for the S&P500 returns, or improve the option pricing model we have used. Also, extending the analysis to more investment horizons, more budget levels, and more option maturities may be a useful sensitivity analysis on our results. Moreover, tail risk protection is seldom bought through a single put position, but rather through more sophisticated approaches, such as a bear spread: assuming tail risk hedges are carried out via these more complex structures can probably deliver a more precise picture on the actual behavior of the strategies we have considered. Further, the performance of the dynamic strategies may be better assessed by studying more sophisticated monetization rules. Finally, extending the number of simulations may probably help acquiring insights on the alleged overpricing of tail risk protection from a standpoint which is different from the usual historical approach adopted in studies of this kind.

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Tables

	STATIC PURCHASE				STATIC SALE				DYNAMIC PURCHASE				DYNAMIC SALE			
	Budget 50, Option 3 months				Budget 50, Option 3 months				Budget 50, Option 3 months, Multiple 0.5				Budget 50, Option 3 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,043	0,156	-1,655	11,676	0,052	0,157	-1,649	11,604	0,044	0,129	-0,345	3,410	0,067	0,184	-1,643	8,528
BuyAndHold	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*
									Budget 50, Option 3 months, Multiple 5				Budget 50, Option 3 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									0,047	0,128	-0,501	3,323	0,061	0,174	-1,345	6,688
BuyAndHold									*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*
	Budget 50, Option 12 months				Budget 50, Option 12 months				Budget 50, Option 12 months, Multiple 0.5				Budget 50, Option 12 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,043	0,158	-1,716	12,717	0,053	0,157	-1,701	12,582	-	-	-	-	-	-	-	-
BuyAndHold	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*	-	-	-	-	-	-	-	-
									Budget 50, Option 12 months, Multiple 5				Budget 50, Option 12 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									-	-	-	-	-	-	-	-
BuyAndHold									-	-	-	-	-	-	-	-
	Budget 150, Option 3 months				Budget 150, Option 3 months				Budget 150, Option 3 months, Multiple 0.5				Budget 150, Option 3 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,037	0,157	-1,638	11,527	0,055	0,156	-1,651	11,624	0,056	0,126	-0,315	3,377	0,080	0,194	-1,330	6,347
BuyAndHold	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*
									Budget 150, Option 3 months, Multiple 5				Budget 150, Option 3 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									0,054	0,122	-0,520	3,382	0,067	0,189	-1,403	6,663
BuyAndHold									*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*
	Budget 150, Option 12 months				Budget 150, Option 12 months				Budget 150, Option 12 months, Multiple 0.5				Budget 150, Option 12 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,033	0,158	-1,716	12,717	0,063	0,155	-1,673	12,326	-	-	-	-	-	-	-	-
BuyAndHold	*0,048*	*0,158*	*-1,716*	*12,717*	*0,048*	*0,158*	*-1,716*	*12,717*	-	-	-	-	-	-	-	-
									Budget 150, Option 12 months, Multiple 5				Budget 150, Option 12 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									-	-	-	-	-	-	-	-
BuyAndHold									-	-	-	-	-	-	-	-

Table 1. First four moments of the strategies over the short-term (1 year)

	STATIC PURCHASE			STATIC SALE			DYNAMIC PURCHASE			DYNAMIC SALE		
	Budget 50, Option 3 months			Budget 50, Option 3 months			Budget 50, Option 3 months, Multiple 0.5			Budget 50, Option 3 months, Multiple 0.5		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,219 (42)	-0,373 (13)	-0,496 (7)	-0,223 (38)	-0,396 (11)	-0,488 (7)	-0,189 (37)	-0,269 (12)	-0,341 (4)	-0,251 (45)	-0,378 (21)	-0,530 (10)
BuyAndHold	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*
							Budget 50, Option 3 months, Multiple 5			Budget 50, Option 3 months, Multiple 5		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,187 (37)	-0,264 (14)	-0,341 (3)	-0,239 (47)	-0,342 (23)	-0,484 (10)
BuyAndHold							*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*
	Budget 50, Option 12 months			Budget 50, Option 12 months			Budget 50, Option 12 months, Multiple 0.5			Budget 50, Option 12 months, Multiple 0.5		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,215 (44)	-0,341 (16)	-0,522 (6)	-0,214 (40)	-0,373 (12)	-0,508 (6)	-	-	-	-	-	-
BuyAndHold	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	-	-	-	-	-	-
							Budget 50, Option 12 months, Multiple 5			Budget 50, Option 12 months, Multiple 5		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-	-	-	-	-	-
BuyAndHold							-	-	-	-	-	-
	Budget 150, Option 3 months			Budget 150, Option 3 months			Budget 150, Option 3 months, Multiple 0.5			Budget 150, Option 3 months, Multiple 0.5		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,221 (44)	-0,326 (19)	-0,506 (7)	-0,216 (39)	-0,391 (11)	-0,483 (7)	-0,172 (36)	-0,278 (8)	-0,315 (3)	-0,256 (47)	-0,373 (23)	-0,485 (12)
BuyAndHold	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*
							Budget 150, Option 3 months, Multiple 5			Budget 150, Option 3 months, Multiple 5		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,174 (35)	-0,268 (10)	-0,322 (2)	-0,270 (45)	-0,372 (24)	-0,485 (12)
BuyAndHold							*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*
	Budget 150, Option 12 months			Budget 150, Option 12 months			Budget 150, Option 12 months, Multiple 0.5			Budget 150, Option 12 months, Multiple 0.5		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,220 (46)	-0,342 (17)	-0,499 (7)	-0,210 (37)	-0,412 (9)	-0,529 (5)	-	-	-	-	-	-
BuyAndHold	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	*-0,212 (43)*	*-0,355 (14)*	*-0,517 (6)*	-	-	-	-	-	-
							Budget 150, Option 12 months, Multiple 5			Budget 150, Option 12 months, Multiple 5		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-	-	-	-	-	-
BuyAndHold							-	-	-	-	-	-

Table 2. Left tail breakdown for the strategies over the short-term (1 year). The values reported are the mean tail losses, i.e. the means of returns worse than -10&, -20% or -30%. The values in brackets are the number of returns worse than -10%, -20% or -30%.

	STATIC PURCHASE				STATIC SALE				DYNAMIC PURCHASE				DYNAMIC SALE			
	Budget 50, Option 3 months				Budget 50, Option 3 months				Budget 50, Option 3 months, Multiple 0.5				Budget 50, Option 3 months, Multiple 0.5			
	Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy	0,209	0,289	-0,401	3,616	0,334	0,280	-0,381	3,478	0,322	0,279	-0,365	3,440	0,232	0,367	-0,700	4,128
BuyAndHold	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
									Budget 50, Option 3 months, Multiple 5				Budget 50, Option 3 months, Multiple 5			
									Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy									0,323	0,278	-0,427	3,697	0,219	0,361	-0,790	4,239
BuyAndHold									*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
	Budget 50, Option 12 months				Budget 50, Option 12 months				Budget 50, Option 12 months, Multiple 0.5				Budget 50, Option 12 months, Multiple 0.5			
	Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy	0,208	0,289	-0,383	3,428	0,321	0,278	-0,398	3,426	0,338	0,260	-0,233	3,779	0,294	0,398	-1,226	5,358
BuyAndHold	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
									Budget 50, Option 12 months, Multiple 5				Budget 50, Option 12 months, Multiple 5			
									Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy									0,302	0,262	-0,554	3,770	0,269	0,377	-1,197	5,643
BuyAndHold									*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
	Budget 150, Option 3 months				Budget 150, Option 3 months				Budget 150, Option 3 months, Multiple 0.5				Budget 150, Option 3 months, Multiple 0.5			
	Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy	0,176	0,289	-0,409	3,651	0,375	0,275	-0,367	3,390	0,361	0,275	-0,384	3,587	0,281	0,401	-0,679	3,890
BuyAndHold	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
									Budget 150, Option 3 months, Multiple 5				Budget 150, Option 3 months, Multiple 5			
									Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy									0,336	0,266	-0,390	3,518	0,257	0,378	-0,684	4,091
BuyAndHold									*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
	Budget 150, Option 12 months				Budget 150, Option 12 months				Budget 150, Option 12 months, Multiple 0.5				Budget 150, Option 12 months, Multiple 0.5			
	Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy	0,160	0,290	-0,385	3,436	0,370	0,271	-0,396	3,393	0,363	0,256	-0,305	3,696	0,358	0,438	-1,130	4,910
BuyAndHold	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*
									Budget 150, Option 12 months, Multiple 5				Budget 150, Option 12 months, Multiple 5			
									Mean	**St. Dev.**	**Skew**	**Exc. Kurt.**	**Mean**	**St. Dev.**	**Skew**	**Exc. Kurt.**
Strategy									0,306	0,243	-0,429	3,817	0,306	0,407	-1,083	4,947
BuyAndHold									*0,233*	*0,290*	*-0,390*	*3,449*	*0,233*	*0,290*	*-0,390*	*3,449*

Table 3. First four moments of the strategies over the medium-term (5 years).

	STATIC PURCHASE			STATIC SALE			DYNAMIC PURCHASE			DYNAMIC SALE		
	Budget 50, Option 3 months			**Budget 50, Option 3 months**			**Budget 50, Option 3 months, Multiple 0.5**			**Budget 50, Option 3 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,267 (42)	-0,373 (22)	-0,502 (12)	-0,303 (15)	-0,398 (9)	-0,476 (6)	-0,275 (18)	-0,403 (9)	-0,493 (6)	-0,343 (53)	-0,438 (36)	-0,556 (23)
BuyAndHold	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
							Budget 50, Option 3 months, Multiple 5			**Budget 50, Option 3 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,302 (17)	-0,383 (11)	-0,510 (6)	-0,351 (54)	-0,464 (35)	-0,575 (23)
BuyAndHold							*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
	Budget 50, Option 12 months			**Budget 50, Option 12 months**			**Budget 50, Option 12 months, Multiple 0.5**			**Budget 50, Option 12 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,269 (43)	-0,367 (24)	-0,494 (12)	-0,262 (21)	-0,365 (11)	-0,483 (5)	-0,232 (14)	-0,398 (5)	-0,504 (3)	-0,492 (37)	-0,558 (31)	-0,664 (23)
BuyAndHold	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
							Budget 50, Option 12 months, Multiple 5			**Budget 50, Option 12 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,268 (21)	-0,343 (13)	-0,446 (7)	-0,457 (38)	-0,513 (32)	-0,637 (22)
BuyAndHold							*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
	Budget 150, Option 3 months			**Budget 150, Option 3 months**			**Budget 150, Option 3 months, Multiple 0.5**			**Budget 150, Option 3 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,275 (48)	-0,371 (27)	-0,535 (12)	-0,289 (12)	-0,384 (7)	-0,482 (4)	-0,273 (14)	-0,459 (6)	-0,459 (6)	-0,367 (50)	-0,468 (34)	-0,587 (22)
BuyAndHold	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
							Budget 150, Option 3 months, Multiple 5			**Budget 150, Option 3 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,237 (18)	-0,437 (6)	-0,472 (5)	-0,343 (51)	-0,472 (31)	-0,581 (21)
BuyAndHold							*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
	Budget 150, Option 12 months			**Budget 150, Option 12 months**			**Budget 150, Option 12 months, Multiple 0.5**			**Budget 150, Option 12 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,275 (55)	-0,362 (33)	-0,491 (16)	-0,251 (15)	-0,343 (8)	-0,509 (3)	-0,248 (10)	-0,390 (4)	-0,551 (2)	-0,500 (37)	-0,582 (30)	-0,687 (23)
BuyAndHold	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*
							Budget 150, Option 12 months, Multiple 5			**Budget 150, Option 12 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,255 (14)	-0,385 (6)	-0,460 (4)	-0,489 (36)	-0,557 (30)	-0,650 (23)
BuyAndHold							*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*	*-0,266 (38)*	*-0,378 (20)*	*-0,476 (12)*

Table 4. Left tail breakdown for the strategies over the medium-term (5 years). The values reported are the mean tail losses, i.e. the means of returns worse than -10&, -20% or -30%. The values in brackets are the number of returns worse than -10%, -20% or -30%.

	STATIC PURCHASE				STATIC SALE				DYNAMIC PURCHASE				DYNAMIC SALE			
	Budget 50, Option 3 months				Budget 50, Option 3 months				Budget 50, Option 3 months, Multiple 0.5				Budget 50, Option 3 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,793	0,407	-0,300	3,139	1,020	0,424	-0,313	3,206	1,029	0,442	-0,385	3,442	0,599	0,634	-0,797	4,390
BuyAndHold	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
									Budget 50, Option 3 months, Multiple 5				Budget 50, Option 3 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									1,019	0,438	-0,416	3,475	0,595	0,611	-0,762	4,279
BuyAndHold									*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
	Budget 50, Option 12 months				Budget 50, Option 12 months				Budget 50, Option 12 months, Multiple 0.5				Budget 50, Option 12 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,672	0,455	-0,350	3,400	0,864	0,440	-0,380	3,566	1,166	0,435	-0,344	3,341	0,685	0,692	-1,020	4,948
BuyAndHold	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
									Budget 50, Option 12 months, Multiple 5				Budget 50, Option 12 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									1,060	0,439	-0,391	3,303	0,654	0,691	-1,384	7,271
BuyAndHold									*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
	Budget 150, Option 3 months				Budget 150, Option 3 months				Budget 150, Option 3 months, Multiple 0.5				Budget 150, Option 3 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,742	0,385	-0,273	3,106	1,148	0,402	-0,254	3,060	1,117	0,434	-0,376	3,400	0,755	0,679	-0,905	5,000
BuyAndHold	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
									Budget 150, Option 3 months, Multiple 5				Budget 150, Option 3 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									1,036	0,422	-0,377	3,463	0,715	0,635	-0,715	4,270
BuyAndHold									*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
	Budget 150, Option 12 months				Budget 150, Option 12 months				Budget 150, Option 12 months, Multiple 0.5				Budget 150, Option 12 months, Multiple 0.5			
	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy	0,581	0,445	-0,350	3,461	0,947	0,430	-0,387	3,595	1,199	0,432	-0,428	3,502	0,835	0,788	-1,672	10,415
BuyAndHold	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*
									Budget 150, Option 12 months, Multiple 5				Budget 150, Option 12 months, Multiple 5			
									Mean	St. Dev.	Skew	Exc. Kurt.	Mean	St. Dev.	Skew	Exc. Kurt.
Strategy									1,021	0,428	-0,397	3,039	0,779	0,715	-0,920	4,278
BuyAndHold									*0,715*	*0,462*	*-0,412*	*3,459*	*0,715*	*0,462*	*-0,412*	*3,459*

Table 5. First four moments of the strategies over the long-term (15 years).

	STATIC PURCHASE			STATIC SALE			DYNAMIC PURCHASE			DYNAMIC SALE		
	Budget 50, Option 3 months			**Budget 50, Option 3 months**			**Budget 50, Option 3 months, Multiple 0.5**			**Budget 50, Option 3 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,329 (6)	-0,371 (5)	-0,393 (4)	-0,411 (2)	-0,673 (1)	-0,673 (1)	-0,551 (2)	-0,551 (2)	-0,551 (2)	-0,596 (36)	-0,697 (29)	-0,866 (11)
BuyAndHold	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
							Budget 50, Option 3 months, Multiple 5			**Budget 50, Option 3 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,395 (3)	-0,527 (2)	-0,527 (2)	-0,537 (36)	-0,663 (27)	-0,950 (16)
BuyAndHold							*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
	Budget 50, Option 12 months			**Budget 50, Option 12 months**			**Budget 50, Option 12 months, Multiple 0.5**			**Budget 50, Option 12 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,339 (15)	-0,403 (11)	-0,595 (5)	-0,522 (4)	-0,522 (4)	-0,788 (2)	-0,338 (2)	-0,488 (1)	-0,488 (1)	-0,681 (34)	-0,749 (30)	-0,823 (26)
BuyAndHold	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
							Budget 50, Option 12 months, Multiple 5			**Budget 50, Option 12 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,464 (2)	-0,464 (2)	-0,464 (2)	-0,629 (39)	-0,720 (33)	-0,801 (28)
BuyAndHold							*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
	Budget 150, Option 3 months			**Budget 150, Option 3 months**			**Budget 150, Option 3 months, Multiple 0.5**			**Budget 150, Option 3 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,311 (6)	-0,311 (6)	-0,409 (3)	-0,366 (1)	-0,366 (1)	-0,366 (1)	-0,407 (2)	-0,407 (2)	-0,573 (1)	-0,625 (29)	-0,774 (22)	-0,929 (17)
BuyAndHold	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
							Budget 150, Option 3 months, Multiple 5			**Budget 150, Option 3 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,306 (3)	-0,577 (1)	-0,577 (1)	-0,518 (30)	-0,657 (22)	-0,809 (16)
BuyAndHold							*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
	Budget 150, Option 12 months			**Budget 150, Option 12 months**			**Budget 150, Option 12 months, Multiple 0.5**			**Budget 150, Option 12 months, Multiple 0.5**		
	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%	-10%	-20%	-30%
Strategy	-0,305 (23)	-0,426 (13)	-0,540 (8)	-0,426 (4)	-0,518 (3)	-0,650 (2)	-0,362 (2)	-0,585 (1)	-0,585 (1)	-0,767 (30)	-0,814 (28)	-0,937 (23)
BuyAndHold	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*
							Budget 150, Option 12 months, Multiple 5			**Budget 150, Option 12 months, Multiple 5**		
							-10%	-20%	-30%	-10%	-20%	-30%
Strategy							-0,524 (1)	-0,524 (1)	-0,524 (1)	-0,638 (34)	-0,724 (29)	-0,876 (22)
BuyAndHold							*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*	*-0,338 (14)*	*-0,445 (9)*	*-0,681 (4)*

Table 6. Left tail breakdown for the strategies over the long-term (15 years). The values reported are the mean tail losses, i.e. the means of returns worse than -10&, -20% or -30%. The values in brackets are the number of returns worse than -10%, -20% or -30%.

Plots

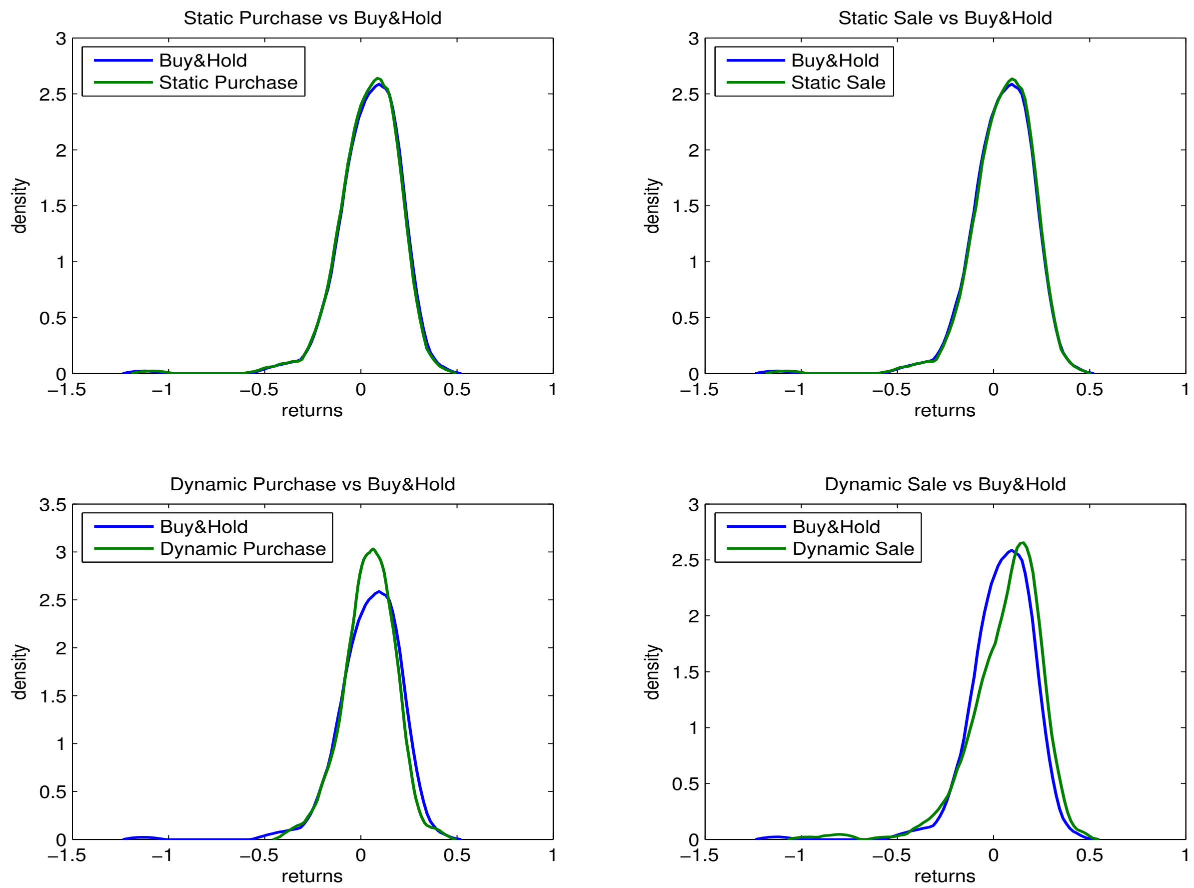

Figure 1. Return distributions of the strategies over the short-term (1 year). Budget = 50bps, option maturity = 3 months, monetization multiple = 0.5.

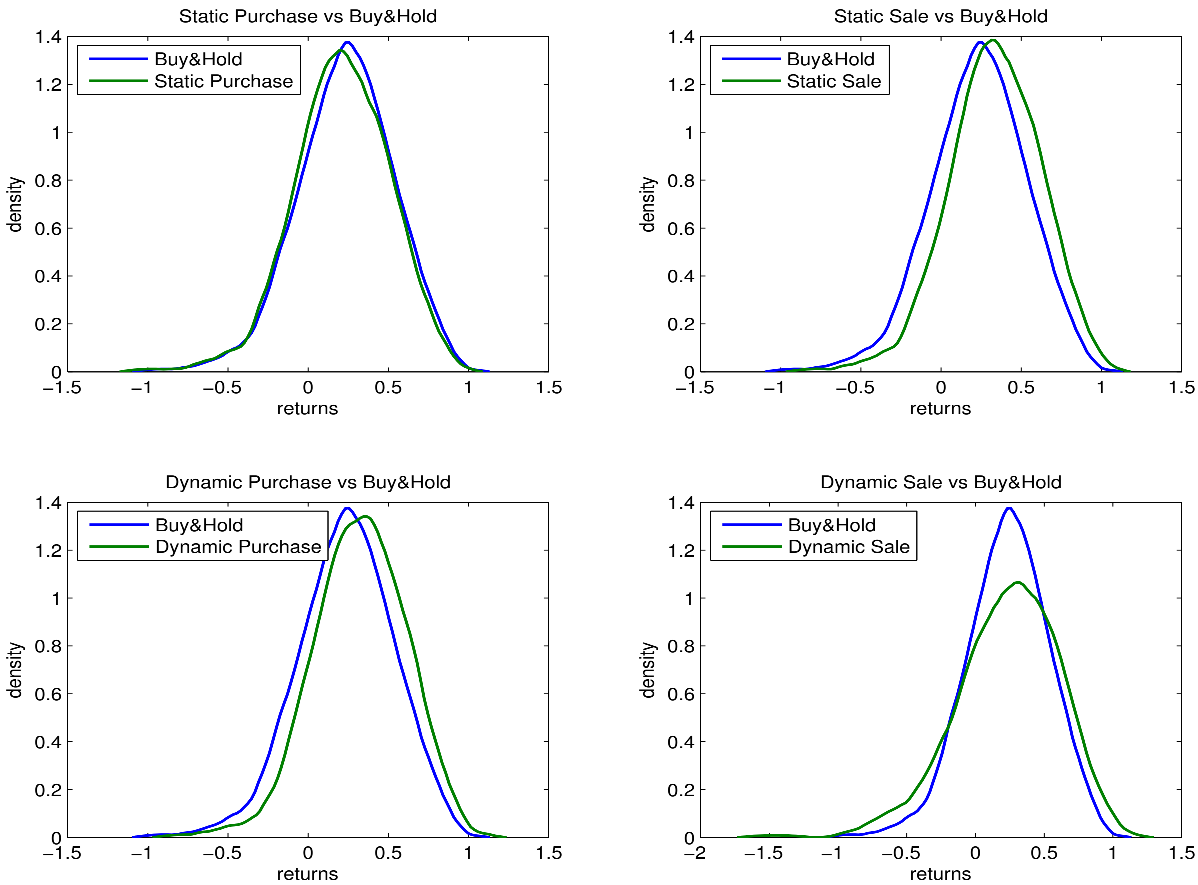

Figure 2. Return distributions of the strategies over the medium-term (5 years). Budget = 50bps, option maturity = 3 months, monetization multiple = 0.5.

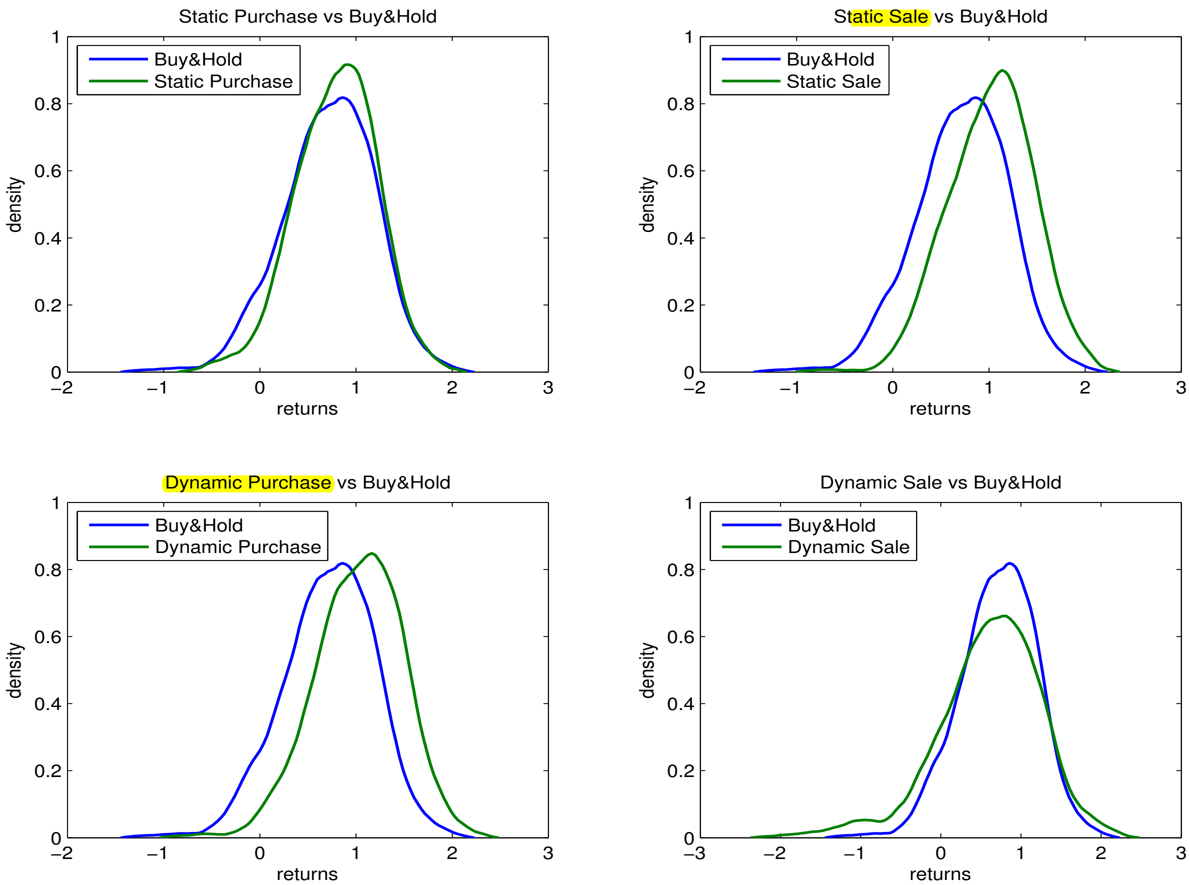

Figure 3. Return distributions of the strategies over the long-term (15 years). Budget = 50bps, option maturity = 3 months, monetization multiple = 0.5.

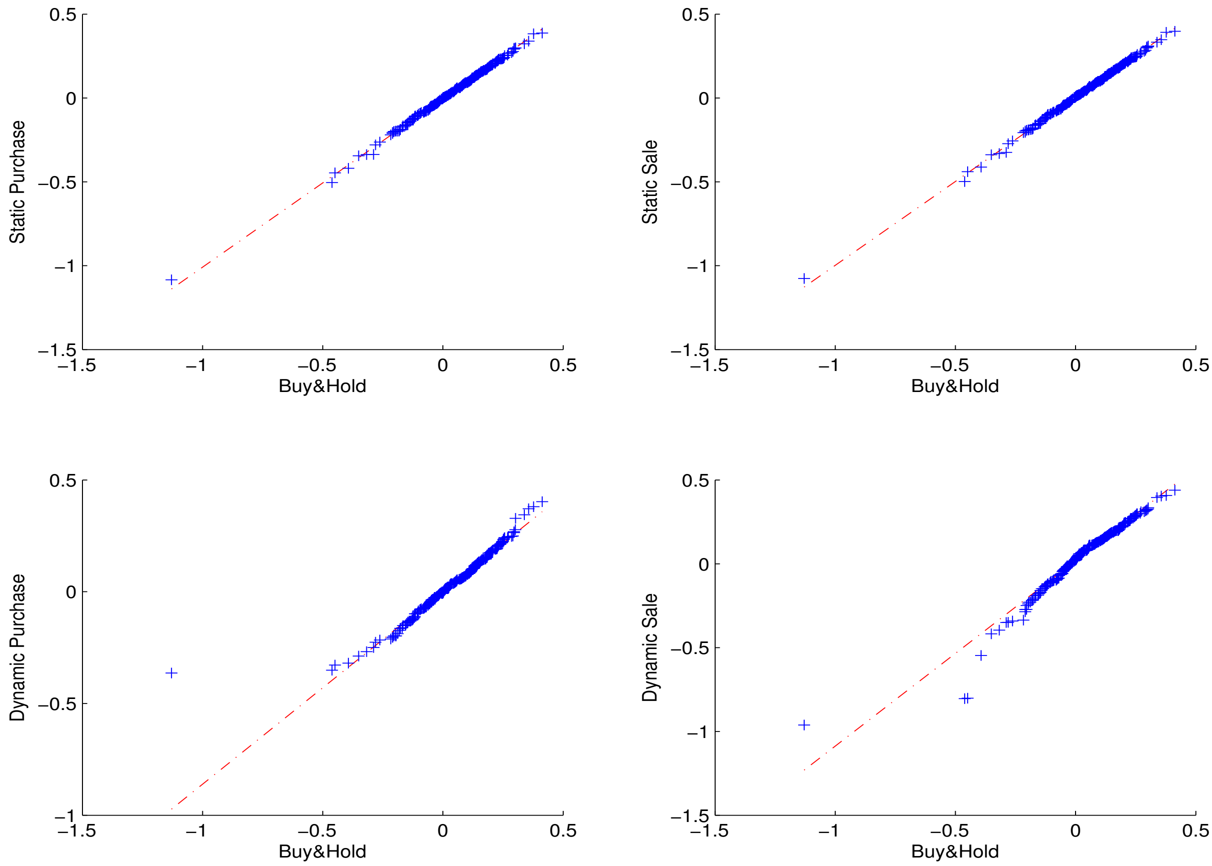

Figure 4. QQ plot for the strategies over the short-term (1 year). Budget = 50bps, option maturity = 3 months, monetization multiple = 0.5.

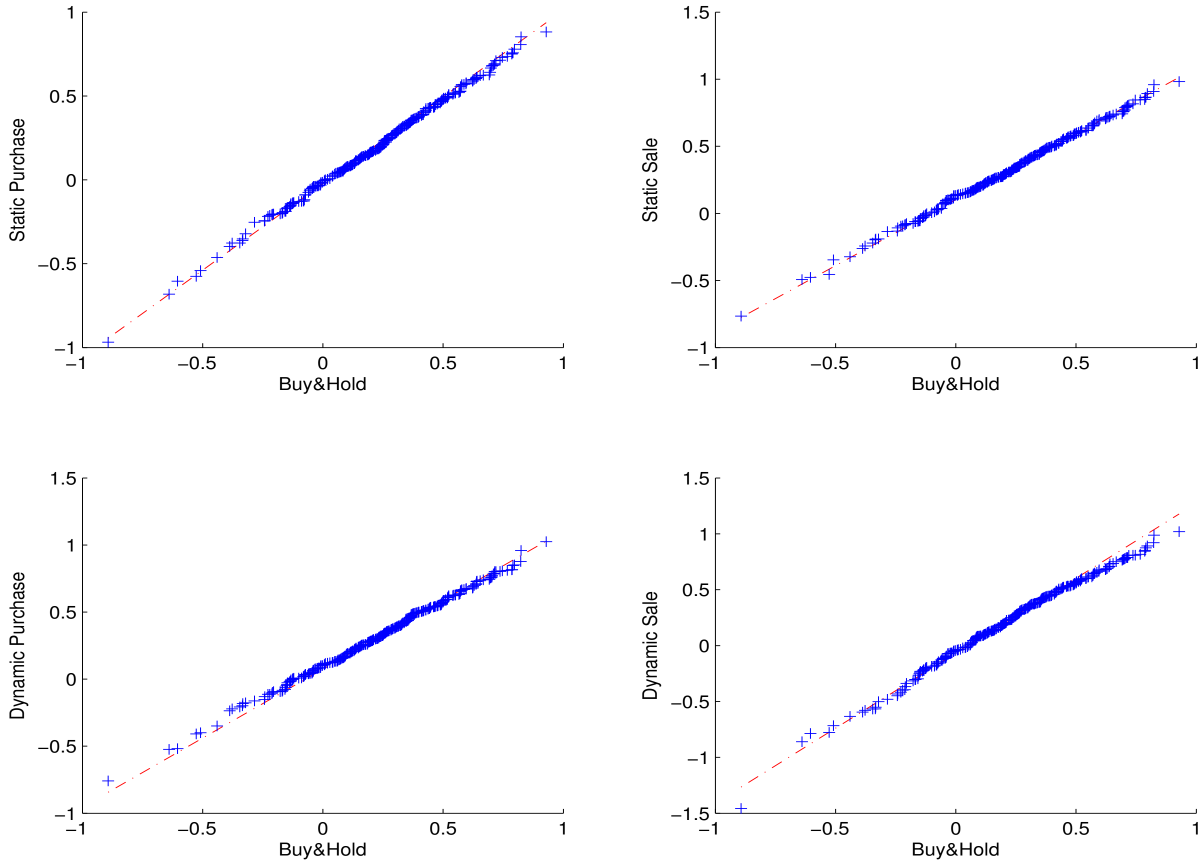

Figure 5. QQ plot for the strategies over the medium-term (5 years). Budget = 50bps, option maturity = 3 months, monetization multiple = 0.5.

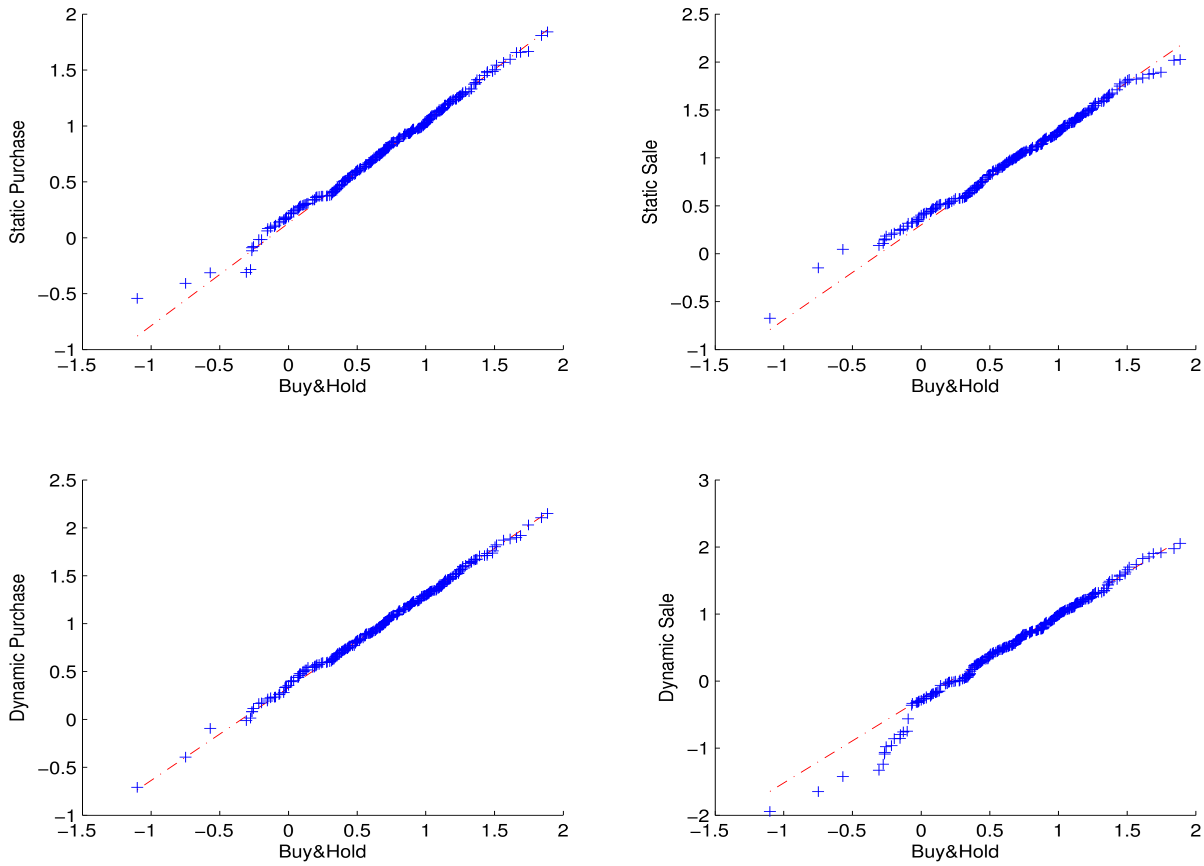

Figure 6. QQ plot for the strategies over the long-term (15 years). Budget = 50bps, option maturity = 3 months, monetization multiple = 0.5.