Volume 15 No. 2 www.iijwm.com Fall 2012

The Journal *of* Wealth Management

"Investment management is a highly fickle discipline. There is plenty of room for successful investors to prosper. Those who do, have learned the need for humility and adopted investment processes which rely on measured decisions and possess discipline."

Jean Brunel - *Editor's Letter*

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"Look at market fluctuations as your friend, rather than your enemy; profit from folly rather than participate in it."

—Warren Buffet

Investors have traditionally equated volatility with risk and viewed it as unavoidable. However, volatility also affects how returns compound over time, which raises the question: Is it possible to profit from volatility? The answer is a definitive yes.

In this article, we explore the concept of *volatility harvesting*, or the extra growth generated from systematically diversifying and rebalancing a portfolio. In contrast to "hunting" for securities with high return potential, we use the term "harvesting" because the activity is akin to farming, where seeds are spread widely and results are patiently harvested over time. The excess return from volatility harvesting is *not* an expected arithmetic excess return derived from forecasting skill, security selection, or an informational advantage. Rather, it is the excess compounded return generated from rebalancing volatile assets over multiple time periods. This excess growth is available in liquid markets with assets that have volatilities greater than zero and correlations less than one. However, only investors with the discipline to trade systematically will harvest this extra growth.

We begin the article with two thought experiments to stimulate the topic and provide insight into the mathematical ideas derived in Appendix. Next, we use market data to evaluate a simple rebalancing strategy for equal-weighted portfolios of stocks in U.S. developed, and emerging markets. We also examine portfolios of stocks selected at random to show that the excess return is independent of an active stock selection process. We show that roughly half of the excess return from volatility harvesting comes from a diversification benefit and half from rebalancing.

We focus on equal weighting because of its simplicity and because it provides a clear illustration of the underlying theory. In practice, a more nuanced approach is required, one that takes into consideration liquidity, trading costs, taxes, and other frictions. In this article, we present theoretical and empirical support for volatility harvesting—the idea that, for assets that are volatile and liquid, diversifying and rebalancing creates excess portfolio growth.

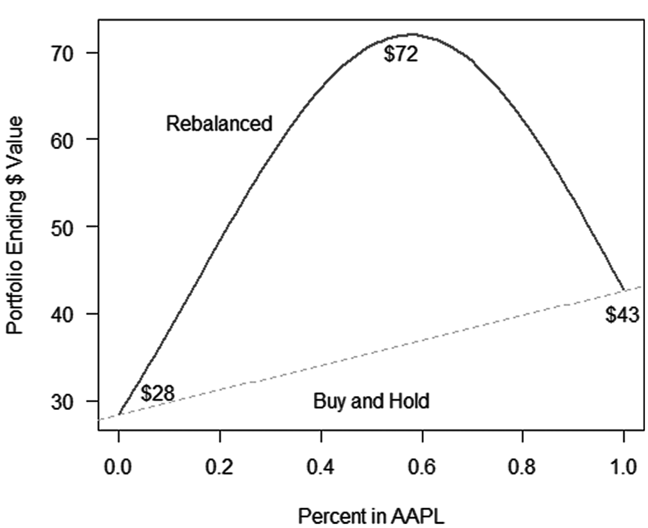
THOUGHT EXPERIMENT 1: APPLE AND STARBUCKS

Assume only two stocks are available for investment: Apple and Starbucks. We selected these two stocks to highlight the value of rebalancing; both are volatile, uncorrelated, and have similar growth rates. Generally, any equity pair in which one stock does not dominate over the entire period will show a benefit from rebalancing. Historically, both Apple and Starbucks were outstanding investments. From 1994 to 2011, a dollar invested in Starbucks grew to $28, and a dollar invested in Apple grew to $43. The annualized growth rates were 21.7% and 24.8%, respectively. A buy-and-hold portfolio that started with half a dollar in each stock would have grown to $35.5, halfway between $28 and $43. For a buy-and-hold investor with foreknowledge of future growth rates, the highest growth portfolio would have been 100% in Apple. However, if the portfolio were rebalanced back to constant weights, then an even higher growth rate could have been achieved. For the rebalancing investor, the best portfolio was 59% Apple and 41% Starbucks. This portfolio would have grown to $72—substantially more than an investment in either stock alone!

Exhibit 1 plots the final wealth for a dollar invested in several different portfolios with weights of 0%, 1%, 2%, and so on, up to 100% in Apple, with the remainder invested in Starbucks. The dashed line represents the ending balance for the buy-and-hold portfolios, the solid line, for rebalanced portfolios. For portfolios that held both stocks, rebalancing led to a higher growth rate than drifting.[1]

Of course, it is possible for a rebalanced portfolio to underperform a buy-and-hold portfolio. For example,

E X H I B I T 1
Growth of $1 for Apple and Starbucks Portfolios (1994–2011)



in the first four years of our sample from 1994 to 1998, Starbucks had steady positive growth while Apple had steady negative growth. In this subperiod, rebalancing hurt performance relative to the buy-and-hold strategy, which allowed the weight of Starbucks to build up in the portfolio. However, a concentrated portfolio is desirable only if the difference in expected future growth rates is very large relative to the volatility of the securities.[2] In practice, future growth rates are unknown and allowing concentration to build up in a portfolio is undesirable.

Exhibit 2 shows the pattern of cumulative excess return from rebalancing by taking the ratio of portfolio values through time. Values above one indicate that a rebalancing portfolio is outperforming a buy and hold. The first four years fall below one, indicating underperformance. However, over the whole sample, the rebalanced portfolio had two times the growth of the buy-and-hold portfolio. The trend of outperformance by the rebalancing strategy is predicted with remarkable accuracy by a rebalancing premium formula represented by the dashed line (see Equation A-8 in Appendix). In some periods, the rebalancing premium is less than theory predicts, and other times, it is greater.

Apple and Starbucks both have high volatilities—50% and 43%, respectively—and a correlation of 0.27. The high volatility and low correlation between these stocks provides a dramatic illustration of the extra growth that can be produced by volatility harvesting. In the next experiment, we show that the extra growth is not only a stock phenomenon but is general to any investment facing uncertainty.

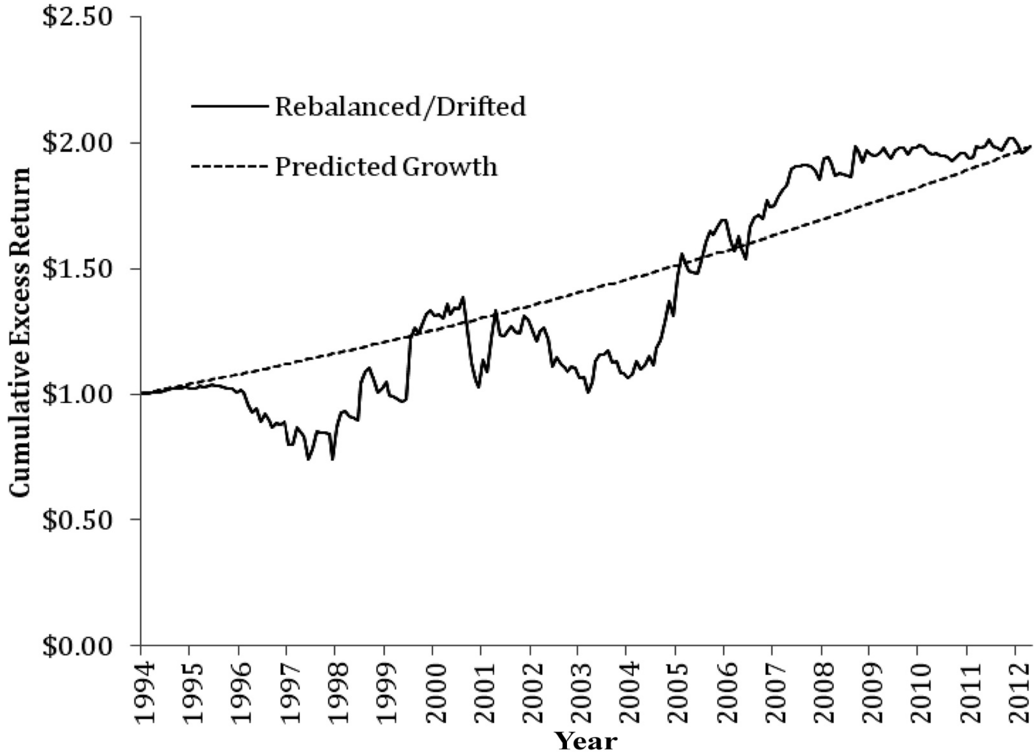
THOUGHT EXPERIMENT 2: COIN FLIPPING FOR FUN AND PROFIT

Imagine a game of chance that depends on the flip of a coin: Heads, and the player can double her money; tails, she loses half. The expected return of a single flip is 25%. If the coin is flipped twice with one result heads and one tails, then the final compounded amount of wealth is the same as the initial amount—zero percent return.

If all proceeds are reinvested and the coin is flipped multiple times, then a run of good luck can earn a lot of money. Ten heads in a row will turn $100 into $102,400. Ten tails in a row makes $100 turn into a few pennies. This game would be attractive to gamblers, since the expected return for a single flip is positive and the

EXHIBIT 2

Cumulative Rebalancing Premium of an Equal-Weighted Portfolio of Apple and Starbucks (1994–2011)



wealth distribution has a highly positive skew (you can win a lot, but you can lose only the initial investment). However, for a risk-averse investor, the game is not very attractive. With a sufficiently large number of flips, the expectation is an equal number of heads and tails. Thus, the game has zero long-term expected growth.

Now imagine that a player always holds half of her money in reserve: Heads, and she can place a portion of the proceeds into her pocket; tails, and she can replenish her stake. Half of her money is always at risk. If the coin is flipped twice, with one result heads and one tails, then this player will earn a 12.5% return. For example, if she put $50 at risk and $50 in her pocket, her risk money would double after the first flip and $25 would be put back in her pocket, so that $75 would be at risk and $75 would be safe. The subsequent loss would apply only to half the assets, leaving $112.50 at the end of two flips. This strategy has a lower expected return on a single flip—12.5% instead of the 25% for the full-investment case—but over many flips, it has a higher long-term growth rate, about 6% on average.[3]

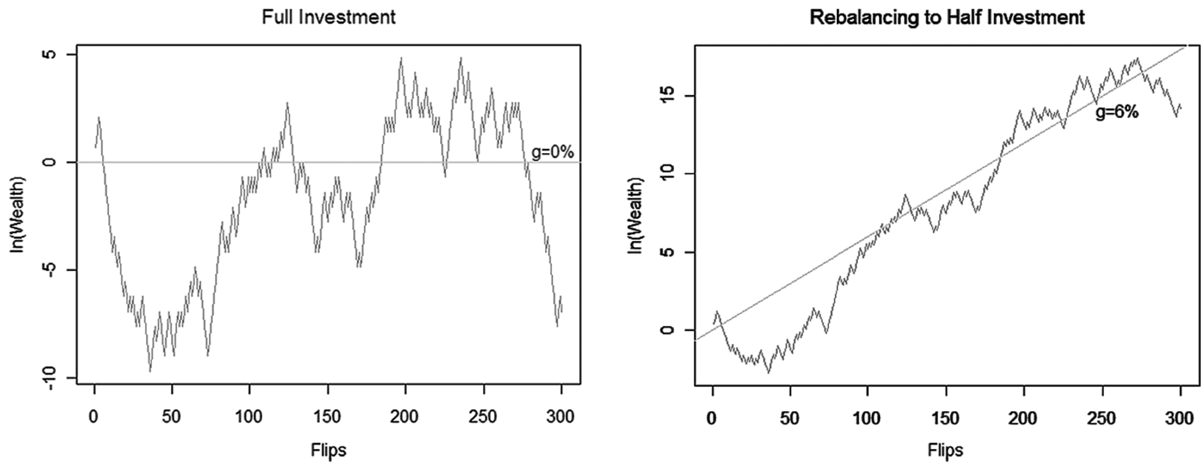
Exhibit 3 shows a simulated return path for random coin tosses in the full-investment and half-investment cases (both strategies using the same sequence of flips). For this game, the wealth levels can quickly get very large or very small, so the charts are plotted on a logarithmic scale in order to observe the growth patterns more clearly.

Each asset—the risky game and the riskless pocket—has zero long-term expected growth. However, the act of trading in the presence of volatility creates a positive growth rate. This is a surprising result when first encountered: Rebalancing creates growth out of no growth, thereby harvesting an expected return from volatility.[4]

INTUITION: WHY DOES REBALANCING WORK?

The above examples illustrate how rebalancing can improve returns, but to understand why it works we must turn to capital growth theory.[5] The literature in this area tends to be quite mathematical, but some intuition can be gained by examining the formulas. One of

EXHIBIT 3
Wealth Simulation for Coin-Flipping Example



the most basic findings is that, for any asset, the growth rate is lower than the average return because volatility is a drag on the compounding effect. Mathematically, the growth of $1 return is given by the expected arithmetic return minus one-half the variance:

$$g = \mu - \frac{\sigma^2}{2} \tag{1}$$

This relationship applies to continuously compounded returns that follow a normal distribution, but is also approximately true for discrete compounding intervals and non-normal returns. For example, an investment with 10% volatility faces a drag on return of 0.50% (since $0.1^2/2 = 0.005$). However, because the function is exponential, the drag grows quickly as volatility increases. Investments with 20%, 40%, and 60% volatilities create a 2%, 8%, and 18% drag per year, respectively. In Appendix, we extend the volatility drag formula to a portfolio of securities with weights, w_i, and derive the following relationship for portfolio growth:

$$\begin{aligned} g_p &= \sum_{i=1}^{N} w_i g_i + \frac{1}{2}\left(\sum_{i=1}^{N} w_i \sigma_i^2 - \sum_{i,j=1}^{N,M} w_i \sigma_{ij} w_j\right) \\ &= \text{Average Growth} + \tfrac{1}{2}\,\text{Average Variance} \\ &\quad - \tfrac{1}{2}\,\text{Portfolio Variance} \\ &= \text{Average Growth} + \text{Rebalancing Premium} \end{aligned} \tag{2}$$

This equation has been described as the "diversification return" by Booth and Fama [1992] and the "rebalancing premium" by Stein, Nemtchinov, and Pittman [2009]. Willenbrock [2011] observes that if diversification is the only "free lunch" of investing, then the diversification return from rebalancing is the only "free dessert." Indeed, rebalancing is closely linked to diversification. A buy-and-hold portfolio, although initially diversified, can drift and become a more concentrated portfolio over time. The control over portfolio concentration, plus the extra growth, makes a strong case for rebalancing.

Manufacturing return out of thin air seems too good to be true. Where do these extra returns come from? There are two distinct components, which we validate empirically in the next section: extra return from diversification and extra return from rebalancing. The diversification return is due to reweighting the portfolio's long-term exposures. For example, an equal-weighted portfolio has less weight in large-cap stocks and more weight in small-cap stocks than the market-cap index. Thus, it has a natural small-cap bias. If small-cap stocks outperform, then an equal-weighted strategy will benefit. However, in addition to creating long-term exposures, another way to earn return is through a pattern of trading. If you can consistently buy low and sell high, you can create positive portfolio growth, even if the overall asset growth is flat.[6]

Intuitively, it is easy to see that trending hurts a rebalancing strategy and large reversals help.[7] Rebalancing, however, is not merely related to momentum and reversal. A deeper reason exists for the observed outperformance. The coin-flipping thought experiment points this out. In that scenario, there is no concept of momentum or reversal since there is no serial correlation between the returns. The probability distribution at each point in time is identical and independent from what has occurred in the past. However, there is still "mean reversion" in this example. If 10 tails in a row are flipped, the mean return is –50%. The sample mean will "revert" to the long-term average as more flips are made. Even in this simple case of no serial correlation, the rebalancing premium is evident. In the more complex case using actual returns, there may be time dependencies. If asset prices experience "boom" periods followed by "bust" periods, then rebalancing will be even more valuable than theory predicts.

Who is on the other side of the trade? The "other trader" is someone who buys after prices have gone up and sells after prices have gone down. There are a few possibilities:

- An investor who chases positive performance (greed) but becomes risk averse when returns become negative (fear);
- A quantitative trader who uses momentum to predict returns;
- An investment manager who prefers "winners" to "losers";
- An institution seeking downside protection through a dynamic "portfolio insurance" trading strategy;
- An institution that receives new cash to invest in good economic times but requires liquidity from its portfolio in bad economic times;

It is hard to imagine the rebalancing premium being arbitraged away. If many investors in the market switched to an equal-weight and rebalance scheme, then volatility in the market would be suppressed. If technology stocks started to boom, investors would sell some of their shares and inhibit price growth. If bank stocks started to crash, investors would step in and buy, supporting the price. Even in this type of market, there would be imbalances in supply and demand, or other exogenous shocks to the system that create volatility and opportunities for rebalancing.

We suspect that the lack of rebalancing is the reason that the capitalization-weighted index underperforms a broad range of portfolio diversification strategies—equal weighting, minimum variance, mean variance, fundamental weighting, diversity weighting, maximum diversity, and others. Chow et al. [2011] observe that, on average, each of the approaches studied outperforms by 1% to 2% per year over 45 years in the U.S. stock market and by a similar amount over 22 years in the global equity markets. Part of these returns can be explained by exposure to value, size, and momentum effects, but there is still a residual excess return. Despite different approaches to portfolio construction, all these strategies have one thing in common: They systematically rebalance. This is likely the source of their residual return.

The key idea is that the growth of a portfolio is the weighted-average growth of the securities plus a rebalancing premium. This premium is always positive because the portfolio variance is always lower than the weighted-average variance of the individual assets when correlations are less than one. Higher volatility and lower correlation among the assets will lead to a higher rebalancing premium.

Another interesting insight to be gained from this formula is that concentrated portfolios will produce a smaller rebalancing premium. For example, if the fixed weights are 99% and 1%, then the rebalancing premium will be lower because only a small portion of the portfolio is being rebalanced. The shape of the plot in Exhibit 1 reveals this result. The most diverse portfolio, equal weight, is not always the highest growth, but it is often close to the highest growth (Platen and Rendek [2010]). Portfolios with more assets and more evenly distributed weights should garner a higher benefit from rebalancing.

Overall, diversifying and rebalancing is a valuable discipline and can be used to exploit volatility. Theoretically, rebalancing reduces concentration risk, downside risk, and volatility, while increasing the long-term growth rate of the portfolio. In practice, it creates a contrarian trading pattern that trades against natural investor tendencies and takes advantage of volatility, reversals, and other return characteristics.

EMPIRICAL EVIDENCE

How does theory hold up when we use actual market returns? Rebalancing is often thought of in the context of asset allocation, particularly with respect to the relative weights of stocks and bonds. A recent paper by Anderson, Bianchi, and Goldberg [2012] compared risk-parity, fixed-weight, and capitalization-weighted portfolios of stocks and bonds. One of their results was that a rebalanced constant mix of 60% stocks and 40% bonds, after transaction costs, outperformed a buy-and-hold mix by 74 basis points per year from 1926 to 2010, with significantly lower volatility.

It also outperformed during each of the subperiods examined: pre-1946, the post-war period between 1946 and 1982, the bull market between 1983 and 2000, and from 2001 to 2010. The capitalization-weighted strategy had an average weight of stocks of 68%. During bull markets, the allocation to equity drifted as high as 95%, and in bear markets, down as low as 30%. It had a higher volatility, suffered more during the crashes, and benefited less during recoveries, because it became highly concentrated. The fixed-weight strategy—holding the weights at 60% stocks, 40% bonds—was more risk controlled. It pulled money out during bull markets and put money in after bear markets (in the same way as in the coin-flipping example) and thus created a higher growth rate. So even in the two-asset case, stocks and bonds, there is return to be created by rebalancing.

Diversification and rebalancing can also be applied at the investment manager, country, industry, or security level. As granularity and volatility increase, the potential for excess growth increases. As a simple empirical test, we examine stock portfolios using monthly historical data from the Russell Global Index, which extends from January 1997 to March 2012.[8] Exhibit 4 shows the performance of global, U.S., developed ex-U.S., and emerging market stock portfolios. For each region, three strategies are tested: capitalization weighted (CAP), equal weights allowed to drift with no rebalancing (EWD), and equal weights rebalanced monthly (EWR).[9] Differences between CAP and EWD capture the benefit of diversification, while differences between EWD and EWR capture the rebalancing premium. Diversification, in this context, refers to the lack of concentration in the portfolio weights.

Over this 15-year period, the EWD portfolios tended to outperform the CAP, except in emerging markets where their performances were approximately equal. However, of greater interest is the additional return generated by rebalancing the equal-weighted portfolio. For example, for the U.S. market, EWD resulted in a 0.26% outperformance over CAP, while the rebalancing to equal weights earned a 1.68% outperformance. The difference of 1.42% is the rebalancing premium. The rebalancing premiums for global, developed, and emerging markets were 0.72%, 0.34%, and 1.41%, respectively.

EXHIBIT 4

Characteristics of Stock Portfolios (January 1997–March 2012) for CAP, EWD, and EWR

Strategy	Total Return	Volatility	Return/ Volatility	Excess Return	Tracking Error	Information Ratio	Turnover One-Way
Global							
CAP	6.31	17.20	0.37				13.75
EWD	7.65	19.27	0.40	1.34	5.07	0.30	19.23
EWR	8.37	19.64	0.43	2.06	5.66	0.40	60.59
U.S. 3000							
CAP	6.47	16.86	0.38				11.50
EWD	6.73	20.81	0.32	0.26	8.84	0.07	21.66
EWR	8.15	22.61	0.36	1.68	10.06	0.22	72.44
Dev. Ex-U.S.							
CAP	5.17	17.96	0.29				17.27
EWD	5.90	18.50	0.32	0.73	3.94	0.19	19.15
EWR	6.24	24.18	0.26	1.07	5.03	0.22	56.51
Emerging							
CAP	7.71	26.98	0.29				29.91
EWD	7.63	26.81	0.28	–0.08	6.03	0.16	30.28
EWR	9.04	28.20	0.32	1.33	7.63	0.16	78.97

In these examples, volatility is higher for the equal-weighted portfolios. This is true for most equity portfolios, as stocks tend to be highly correlated with one another and smaller stocks tend to be more volatile. In asset classes with lower cross-correlations, equal-weighted portfolios are less volatile than the more concentrated indexes. For example, an equal-weighted portfolio of commodities has a lower volatility than its index because commodities have lower cross-correlations and the indexes are concentrated in highly volatile energy contracts.

As expected, turnover is higher in the rebalancing portfolios. In practice, turnover and liquidity issues would need to be addressed to ensure that benefits from rebalancing are not eroded by trading frictions. Some possible ways to address these issues are:

- To reduce the frequency of rebalancing, allow the portfolio to drift within specified bounds
- To reduce the overall amount of trading, rebalance at the country or sector level instead of the stock level
- To alleviate liquidity issues, use a diversification function to create weights that are a compromise between capitalization weights and equal weights

Despite the strong theoretical support, numerous engineering problems need to be addressed in a real portfolio.

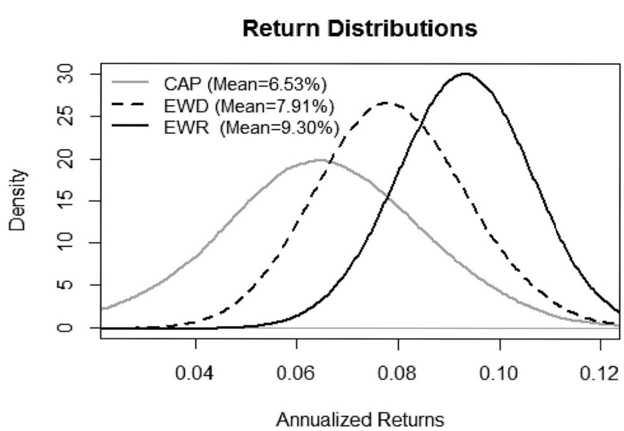
As a final empirical illustration, we examine simulated portfolios of random stocks. Simulation provides an avenue to isolate diversification and rebalancing premia while controlling for selection and weighting effects. For a large number of trials, we selected 100 stocks at random from the Russell Global Index, thus simulating an active stock selection strategy. Using the same time period, we again examine three strategies: capitalization weighted (CAP), equal weights allowed to drift with no rebalancing (EWD), and equal weights rebalanced monthly (EWR).[10] Each trial holds the same 100 stocks across portfolio construction strategies. As before, the difference between CAP and EWD captures the benefit of diversification, while the difference between EWD and EWR captures the rebalancing premium.

The first panel of Exhibit 5 illustrates the distribution of simulated annualized return outcomes by strategy. On average, EWR contributes almost 2.80% of annual excess return above CAP, and EWD posts nearly a 1.40% return improvement. Thus, we can attribute the excess annual return in almost equal parts to diversification and rebalancing.

The second panel of Exhibit 5 depicts the distribution of annualized volatility outcomes by strategy. Both the level and the range of annualized volatility outcomes show meaningful improvement compared with the capitalization-weighted strategy. While all of strategies post an average annualized volatility of almost 20%, allowing weights to drift increases the likelihood

EXHIBIT 5

Return and Volatility Distributions for Random 100-Stock Portfolios by Portfolio Construction Strategy (January 1997–March 2012)



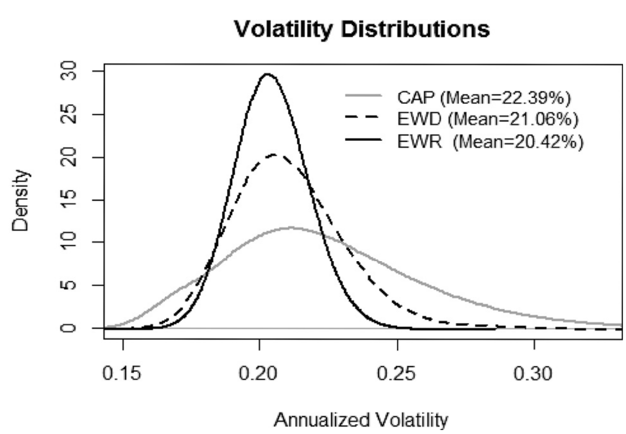
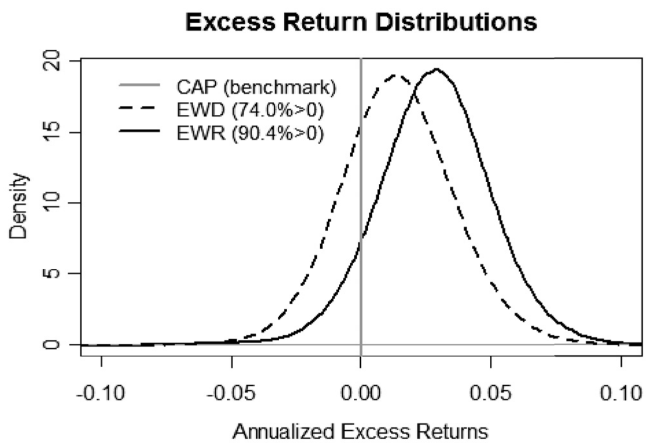


EXHIBIT 6

Excess Returns over Market-Cap Weighting for Random 100-Stock Portfolios (January 1997–March 2012)



of higher-volatility outcomes. Equal weighting significantly decreases the likelihood of outsized high-volatility outcomes in the right tail (in this context the right tail is bad). Monthly rebalancing magnifies this effect by keeping the initial portfolio and future portfolios diversified. In other words, higher concentration can lead to higher risk.

Measuring the excess returns of each strategy within each trial provides additional insight into the nature of diversification and rebalancing effects. Exhibit 6 depicts distributions of returns in excess of the capitalization-weighted strategy. The diversification benefits of EWD lead it to outperform CAP nearly 74% of the time. Monthly rebalancing raises the likelihood of outperformance to over 90% of trials. The symmetric, smooth distribution of excess returns suggests results hold consistently across trials.

When looking at concentrated portfolios of stocks, regardless of which securities are selected, we find that nearly half of the excess return from an equal-weighted portfolio comes from rebalancing. We also find that drifting portfolios tend to experience a buildup in concentration, volatility, and risk.

CONCLUSION

In the end, our advice is simple: diversify and rebalance. This prescription not only provides a framework for managing risk, but also enhances returns in the long term. In a real portfolio, the turnover generated by rebalancing can be costly, particularly when transaction costs are high. Unconstrained rebalancing could result in transaction costs that outweigh the rebalancing benefits. Of course, of key interest to the practitioner is the question of how to reduce these costs and measure how much would be given up in performance. In an upcoming article, we plan to discuss implementation issues and circumstances in which costs can be carefully controlled. Specifically, we will show that a number of pragmatic methods can substantially reduce the frequency of rebalancing and trading costs. This can be achieved, for example, by allowing the portfolio to drift within bounds, by enforcing diversity at the country and sector level, and by selecting weights that seek the right compromise between capitalization weighting and equal weighting.

The principles presented here are mathematical in nature and apply to any set of sufficiently liquid investments that are volatile and uncorrelated; therefore, they can be applied at the asset allocation level and within subsegments of the portfolio. Investors should consider their portfolios in a multiperiod framework and realize that volatility is more than just a risk measure—it represents an opportunity that can be exploited through thoughtful rebalancing. Just as it is possible to harness energy from waves in the ocean, it is possible to harvest return from volatility in the market.

APPENDIX

DERIVATION OF VOLATILITY DRAG AND DIVERSIFICATION RETURN

Following the option-pricing and capital growth literature, we assume that returns follow a geometric Brownian motion process:

$$\frac{dS}{S} = \mu \cdot dt + \sigma \cdot dz \tag{A-1}$$

where S is the asset price, μ is the expected return, σ is the volatility, dt is the time increment, and dz is a normal random variable N(0,1). From Ito's Lemma we know that the above stochastic differential equation has the following solution:

$$dG = \left(\frac{\partial G}{\partial S} \mu S + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{dG}{dS} \sigma S dz \tag{A-2}$$

If we let prices follow a lognormal process, G = ln(S), then:

$$\frac{\partial G}{\partial S}=\frac{1}{S}; \qquad \frac{\partial^2 G}{\partial S^2}=-\frac{1}{S^2}; \qquad \frac{\partial G}{\partial t}=0 \qquad \text{(A-3)}$$

Substituting back into Ito's formula:

$$dG=\left(\mu-\frac{\sigma^2}{2}\right)dt+\sigma dz \qquad \text{(A-4)}$$

Thus, the continuously compounded return $dG = d\ln(S) = dS/S$ is a geometric Brownian motion process with drift parameter:

$$g=\mu-\frac{\sigma^2}{2} \qquad \text{(A-5)}$$

When assets are held within a fixed-weight portfolio, the long-term growth from Equation (A-5) becomes

$$g_p=\sum_{i=1}^{N} w_i\mu_i-\frac{1}{2}\sum_{i,j=1}^{N,M} w_i\sigma_{ij}w_j \qquad \text{(A-6)}$$

Where w_i is the portfolio weight allocated to asset i, σ_{ij} is the return covariance of asset i and j, and g_p is the continuously compounded portfolio return. To emphasize the benefit of diversification on portfolio return, we solve for μ from Equation (A-5) and substitute into Equation (A-6) to obtain

$$g_p=\sum_{i=1}^{N} w_i\left(g_i+\frac{\sigma_i^2}{2}\right)-\frac{1}{2}\sum_{i,j=1}^{N,M} w_i\sigma_{ij}w_j \qquad \text{(A-7)}$$

We can rewrite this as

$$g_p=\sum_{i=1}^{N} w_i g_i+d \qquad \text{(A-8)}$$

where

$$d=\frac{1}{2}\sum_{i=1}^{N} w_i\sigma_i^2-\frac{1}{2}\sum_{i,j=1}^{N,M} w_i\sigma_{ij}w_j \qquad \text{(A-9)}$$

Equation (A-8) expresses the portfolio growth rate as the sum of the individual asset growth rates plus the premium d derived from diversification and rebalancing. This value is positive for correlations less than one, implying that the benefit of rebalancing to fixed weights is positive. The first term of d is the weighted sum of the component asset variances, and the second is the portfolio variance. An increase in asset volatility increases the first term (increases growth potential from rebalancing) but also increases portfolio variance (decreases growth). The amount by which the second term increases relative to the first largely depends on the correlation among the assets.

ENDNOTES

[1]Cover [1991] shows several stock pair examples and generalizes to the concept of a "universal" portfolio. Maslov and Zhang [1998] use an example of rebalancing between cash and a Russian stock with negative growth but with volatility high enough that rebalancing creates positive portfolio growth.

[2]Jamshidian [1992] formalizes this idea.

[3]There is a 50% probability of earning 50% or losing 25%. The expected growth rate for a large number of trials is 0.5*ln(1+0.5)+0.5*ln(1-0.25) = 0.059. For the full investment case, 0.5*ln(1+1.0)+0.5*ln(1-0.5) = 0. See Luenberger [1998], chap. 15.

[4]We can also use simulation to estimate the rebalancing premium for more realistic scenarios. See Stein, Nemtchinov, and Pittman [2008] for an estimate of the rebalancing premium in the emerging markets.

[5]See Maclean, Thorpe, and Ziemba [2011] for a synthesis of capital growth theory.

[6]Plyakha, Uppal, and Vilkov [2012] isolate the rebalancing premium from size, value, and momentum effects. After controlling for these factors, they find significant excess alpha attributed to rebalancing.

[7]Perold and Sharpe [1995] and Wise [1996] discuss dynamic strategies and the intuition behind them.

[8]Other studies have used different datasets and time periods and produced similar results. For example, DiMiguel, Garlappi, and Uppal [2009], Platen and Rendek [2010], and Plyakha, Uppal, and Vilkov [2012] provide empirical support to equal-weight and rebalance strategies.

[9]Note that the capitalization-weighted strategy will have a slightly different return than the official index since it is calculated monthly instead of daily. For example, the Russell 3000 Index calculated on a daily basis produced a 6.51% return over this period. Our monthly calculation came to 6.47%, which is close but not equal to the official index return.

[10]Stocks with data as of January 31, 1997, are used. Over time, as stocks leave the sample, the final weight is reinvested on either a market-capitalization or equal-weighted basis. The simulation drew one million trials of one hundred stocks.

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